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A matrix model for Misner universe and closed string tachyons

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Abstract: We use D-instantons to probe the geometry of Misner universe, and calculate the world volume field theory action, which is of the 1+0 dimensional form and highly non-local. Turning on closed string tachyons, we see from the deformed moduli space of the D-instantons that the spacelike singularity is removed and the region near the singularity becomes a fuzzy cone, where space and time do not commute. When realized cosmologically there can be controllable trans-planckian effects. And the infinite past is now causally connected with the infinite future, thus also providing a model for big crunch/big bang transition. In the spirit of IKKT matrix theory, we propose that the D-instanton action here provides a holographic description for Misner universe and time is generated dynamically. In addition we show that winding string production from the vacua and instability of D-branes have simple uniform interpretations in this second quantized formalism.

Keywords: D-branes, M(atrix) Theories.

Contents

1. Introduction

The resolution of spacelike singularities is one of the most outstanding problems in the study of quantum gravity. These singularities make appearance in many black holes and cosmological models. Unfortunately it is very hard to get much information about them in general situations. So in order to make progress on this issue, more controllable toy models are proposed, the simplest of which may be the two dimensional Misner space, which can be defined as the quotient of two dimensional Minkowski space by a boost transformation.

Nowadays string theory is widely regarded as the most promising candidate for a quantum theory of gravity. And actually string theory does provide resolution for some singularities, such as orbifolds(2.2), conifolds [1] and enhancons [2]. For spacelike singularities, less has been achieved. For example, even the most familiar GR singularity inside the Schwarzschild black hole has not yet been understood.

Misner space can be embedded into string theory by adding 8 additional flat directions, and it is an exact solution of string theory at least at tree-level [3]. The dynamics of particles and strings in Misner universe were much explored in the literature (see for example [4 – 8], for a good review see [9]). In particular, it was realized in the above papers that winding strings are pair-produced and they backreact on the geometry. Hence they may play important role in the resolution of the singularity. Unfortunately, it is fair to say that we still lack a sensible treatment of the backreactions.

Along another line, in the study of closed string tachyons [10 – 12], Misner space has reemerged as a valuable model [13]. By imposing anti-periodic boundary conditions for fermions on the spatial circle, one can get winding tachyons near the singularity which can significantly deform the original geometry. It is argued [13] that the spacetime near the spacelike singularity will be replaced by a new phase of the tachyon condensate. In their case the influence of the winding modes to the spacetime geometry is more significant and more tractable. It is mainly this work [13] that motivates our following study.

We will use D-branes to probe the background geometry. D-branes are attractive here because they can feel distances smaller than string scale [14]. For Misner universe, the singularity is localized in sub-string region in the time direction, so we will use D-instantons as probes. Recently, D0 and D1-branes in Misner space were studied in [15], and it was found that they are both unstable due to open string pair production and closed string emission.

In fact, the usefulness of D-instantons has deeper reasons. It was conjectured in [16] that the large-N limit of the supersymmetric matrix quantum mechanics describing D0-branes provides a holographic description of M-theory in the light cone frame. In this model, known as BFSS matrix theory, all spatial dimensions are dynamically generated while time is put in a prior. Later in [17], another matrix theory is proposed for IIB theory. This so called IKKT matrix theory is a 0+0 dimensional theory, in contrast to the 0+1 dimensional BFSS theory. Thus in this theory, both spatial and temporal dimensions are generated dynamically. In fact, the IKKT action is essentially just the D-instanton action. Thus the D-instanton action provides a holographic description of the full string theory. We will argue in this note that this also happens for D-instantons in the Misner universe.

One of the advantages of the holographic description is that backreaction can be taken into account more naturally, since the geometry and objects in it are not treated seperatedly as in conventional theory including perturbative string theory. And the drawback is that it is often tricky to get detailed information from these matrices. And in this note we will encounter both the advantages and disadvantages.

The layout of this note is as follows. In section 2, we review some aspects of the Misner geometry and properties of closed strings in it. We begin in section 3 the investigation of D-instanton physics. We derive the matrix action, and identify the vacuum corresponding to the background geometry. And in section 4, we study how tachyons affect spacetime. We read from the tachyon deformed D-instanton action the new moduli space, and thus the resulted geometry. Finally in section 5, we promote our probe action to a second quantized framework, which can be regarded as an lorentzian orbifolded version of IKKT matrix theory. We construct D-branes from matrices and study their properties, giving evidence that the D-instanton action actually provides a holographic description for Misner universe.

Recent explorations of other singularities include [39 − 41].

Note added: After our paper was submitted to archive, we received another paper [42] addressing similar problems.

2. Misner universe: the geometry and the closed string story

Misner universe is an orbifold of 1+1-dimensional Minkowski space

$$ds^2 = -2dx^+dx^- \tag{2.1}$$

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by the identification

$$x^+ \sim e^{2\pi\gamma} x^+, \quad x^- \sim e^{-2\pi\gamma} x^-. \tag{2.2}$$

Coordinate transformation

$$x^+ = \frac{T}{\sqrt{2}} e^{\gamma\theta}, \quad x^- = \frac{T}{\sqrt{2}} e^{-\gamma\theta} \tag{2.3}$$

can be made to write Misner space as

$$ds^2 = -dT^2 + \gamma^2 T^2 d\theta^2, \tag{2.4}$$

with $\theta \cong \theta + 2\pi$. It is easy to see from (2.4) that this space-time contains two cosmological regions connected by a space-like singularity.

There are generally two kinds of closed strings in Misner universe: twisted and untwisted. Untwisted states include in particular the gravitons and their behaviors are particle-like. Their wave functions can be obtained by superposing a plane wave in the parent Minkowski space with its images under the boost (2.2), and is written as [4, 7]

$$f_{j,m^2,s}(x^+, x^-) = \int_{-\infty}^{\infty} dv e^{ip^+ X^- e^{-2\pi\gamma v} + ip^- X^+ e^{2\pi\gamma v} + ivj + vs}, \tag{2.5}$$

with j the boost momentum, m the mass, and s the SO(1,1) spin in $R^{(1,1)}$.

Due to the orbifold projection (2.2), new twisted sectors arise in Misner space with strings satisfying

$$X^\pm(\tau, \sigma + 2\pi) = e^{\pm 2\pi\gamma w} X^\pm(\tau, \sigma), \tag{2.6}$$

where the winding number w is an integer. Many mysterieses of the Misner universe have origin from these winding strings. It was shown in [6] that there exists a delta-function normalizable continuum of physical twisted states, which can be pair produced in analogy with the Schwinger effect in an electric field. And evaluating the Bogolubov coefficients, they showed that the transmission coefficient reads

$$q_4 = e^{-\pi M^2/2\nu} \frac{\cosh(\pi \tilde{M}^2/2\nu)}{|\sinh \pi j|}, \tag{2.7}$$

where $\nu = -\gamma w$ is the product of the boost parameter and the winding number, and

$$M^2 = \alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+, \quad \tilde{M}^2 = \tilde{\alpha}_0^+ \tilde{\alpha}_0^- + \tilde{\alpha}_0^- \tilde{\alpha}_0^+ \tag{2.8}$$

with string zero modes $\alpha_0^\pm$ and $\tilde{\alpha}_0^\pm$, comes from the Virasoro conditions.

3. D-instantons probing Misner universe

We embed the geometry (2.1) (2.4) into string theory by adding another 8 flat directions $Y^a, a = 1, \ldots, 8$, and then put N D-instantons in this geometry then go on to find the field theory describing their behavior. We want to read from the modular space of the D-instantons the background geometry, following the study of [10, 18] . In this note we ignore the backreaction of these D-instantons.

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D-brane dynamics on the orbifolds were variously discussed in the previous literature. We follow mainly Taylor's procedure [19]. The open string degrees of freedom form a matrix theory. We focus on the bosonic part, which are the embedding coordinates. Go to the covering space

$$(X^+, X^-) \in R^{1,1}, \quad Y^a \in R^8_\perp, \tag{3.1}$$

and make the projection(2.2), then each D-instanton has infinitely many images, which can be captured by matrices of infinitely many blocks. Each block is itself a $N \times N$ matrix. The orbfold projection for these blocks reads

$$\begin{aligned} X^+_{i,j} &= e^{2\pi\gamma} X^+_{i-1,j-1}, \\ X^-_{i,j} &= e^{-2\pi\gamma} X^-_{i-1,j-1}, \\ Y^a_{i,j} &= Y^a_{i-1,j-1}. \end{aligned} \tag{3.2}$$

These matrices can be solved using the following basis:

$$(\beta^m_l)_{ij} = e^{2\pi i l\gamma} \delta_{i,j-m}. \tag{3.3}$$

Some of their communication relations will be used in this note:

$$\begin{aligned} \left[\beta^m_0, \beta^{m'}_0\right] &= 0 \\ \left[\beta^m_0, \beta^{m'}_1\right] &= (e^{2\pi m\gamma} - 1)\beta^{m+m'}_1 \\ \left[\beta^m_0, \beta^{m'}_{-1}\right] &= (e^{-2\pi m\gamma} - 1)\beta^{m+m'}_{-1} \\ \left[\beta^m_1, \beta^{m'}_{-1}\right] &= (e^{-2\pi m\gamma} - e^{2\pi m'\gamma})\beta^{m+m'}_0. \end{aligned} \tag{3.4}$$

The solutions thus read

$$\begin{aligned} X^+ &= \sum_{m\in Z} x^+_m \beta^m_1, \\ X^- &= \sum_{m\in Z} x^-_m \beta^m_{-1}, \\ Y^a &= \sum_{m\in Z} y^a_m \beta^m_0. \end{aligned} \tag{3.5}$$

The low energy effective action for the D-instantons can be obtained from dimensional reduction of 10-d Super Yang-Mills, and keep only the bosonic part, we get

$$S = \frac{1}{2g^2 Z_0} \sum_{\mu,\nu=0}^{9} \mathrm{Tr}\left([X^\mu, X^\nu]\,[X_\mu, X_\nu]\right), \tag{3.6}$$

with coupling $g^2 = \frac{g_s}{\alpha'^2}$, where we eliminate factors of order 1; Z_0 is the normalization factor which is formally trace of the infinite dimensional unite matrix. The above action is written in the Minkowski signature, so there is an overall sign difference with IKKT [17].

Written in terms of the above solution(3.5), the action reads

$$S = -\frac{1}{g^2} \sum_{m+m'+n+n'=0} \mathrm{Tr} \Bigg[x^+_m x^+_{m'} x^-_n x^-_{n'} (e^{-2\pi m\gamma} - e^{2\pi n\gamma})(e^{-2\pi m'\gamma} - e^{2\pi n'\gamma}) + \\ + 2x^+_m x^-_{m'} y^a_n y^a_{n'} (e^{2\pi n\gamma} - 1)(e^{2\pi n'\gamma} - 1) e^{-2\pi(m+n)\gamma} \Bigg]. \quad (3.7)$$

The above action has many branches of vacuum. In the following, we will consider the Higgs branch which corresponds to D-branes probing the Misner part of the geometry, with the same coordinates in the other 8 directions. Thus we can eliminate the second term of the above action.

The infinite summation in (3.7) indicates a "hidden" dimension with topology S^1, on which the Fourier coefficients of a real scalar field can represent the modes in (3.5). That is

$$\begin{aligned} x^+_m &= \int_0^{2\pi} \frac{d\sigma}{\sqrt{2\pi}} X^+(\sigma) e^{-im\sigma}, \\ x^-_m &= \int_0^{2\pi} \frac{d\sigma}{\sqrt{2\pi}} X^-(\sigma) e^{-im\sigma}, \end{aligned} \quad (3.8)$$

and the action

$$\begin{aligned} S &= -\frac{1}{g^2} \int_0^{2\pi} \frac{d\sigma}{2\pi} \mathrm{Tr} \left([e^{i2\pi\gamma \frac{d}{d\sigma}} X^+(\sigma)] X^-(\sigma) - [e^{-i2\pi\gamma \frac{d}{d\sigma}} X^-(\sigma)] X^+(\sigma) \right)^2 \\ &= -\frac{1}{g^2} \int_0^{2\pi} \frac{d\sigma}{2\pi} \mathrm{Tr} \Big[X^+(\sigma + i2\pi\gamma) X^-(\sigma) - X^-(\sigma - i2\pi\gamma) X^+(\sigma) \Big]^2 \end{aligned} \quad (3.9)$$

is complemented by the symmetry

$$\begin{aligned} X^+(\sigma) &\to e^{2\pi\gamma} X^+(\sigma), \\ X^-(\sigma) &\to e^{-2\pi\gamma} X^-(\sigma), \end{aligned} \quad (3.10)$$

inherited from the orbifold projection (2.2).

Note that the system (3.9) (3.10) possesses a large variety of vacua, which are simply solutions of the equation

$$X^+(\sigma + i2\pi\gamma) X^-(\sigma) - X^-(\sigma - i2\pi\gamma) X^+(\sigma) = 0. \quad (3.11)$$

Just to study the space-time geometry, we take all the D-instantons to coincide, and the matrices become ordinary functions. We can define

$$F(\sigma) \equiv X^+(\sigma + i2\pi\gamma) X^-(\sigma), \quad (3.12)$$

which appears repeatedly in this note. And the vacuum condition reads now

$$F(\sigma) = F(\sigma - i2\pi\gamma). \quad (3.13)$$

Note also that X^+, X^- are all defined on a circle, which says

$$F(\sigma) = F(\sigma + 2\pi). \quad (3.14)$$

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For the problems in hand, we expect $X^+(\sigma), X^-(\sigma)$ to have no poles in the σ plane, so $F(\sigma)$ must be a constant.

Thus $X^+(\sigma + i2\pi\gamma)$ can be factorized as a real function of σ with periodicity 2π multiplied by a constant

$$X^+(\sigma + i2\pi\gamma) = \alpha f(\sigma). \tag{3.15}$$

For real functions $X^+(\sigma), f(\sigma)$ with periodicity 2π, we can expand them as

$$X^+(\sigma) = \sum_{n=-\infty}^{+\infty} c_n e^{in\sigma}, \quad f(\sigma) = \sum_{n=-\infty}^{+\infty} f_n e^{in\sigma}, \tag{3.16}$$

with $c_{-n} = c_n^*$ and $f_{-n} = f_n^*$. Then eq. (3.15) leads to

$$c_n e^{-2\pi n\gamma} = \alpha f_n, \tag{3.17}$$

and thus

$$c_{-n} e^{2\pi n\gamma} = \alpha f_{-n}, \tag{3.18}$$

or

$$c_n^* e^{2\pi n\gamma} = \alpha f_n^*, \tag{3.19}$$

and

$$c_n^* e^{-2\pi n\gamma} = \alpha^* f_n^*. \tag{3.20}$$

For any non vanishing c_n, f_n, eq. (3.19) (3.20) require

$$e^{4\pi n\gamma} = \frac{\alpha}{\alpha^*}, \tag{3.21}$$

which obviously can not be satisfied for more than one value of n. And note that for n non-zero, c_n, f_n are paired with c_{-n}, f_{-n}. So all c_n, f_n except c_0, f_0 must vanish, and thus $X^+(\sigma)$ must be a constant, which subsequently forces $X^-(\sigma)$ also to be a constant.

Taking into account the constraint (3.10), we get a branch of the moduli space (the Higgs branch)

$$\mathcal{M} = \left\{ X^+, X^-, Y^a \in R \middle/ \left\{ \begin{array}{l} X^+ \cong e^{2\pi\gamma} X^+ \\ X^- \cong e^{-2\pi\gamma} X^- \end{array} \right\} \right\} \tag{3.22}$$

which is exactly the original Misner universe.

To end this section, we remind the reader of some characteristics of the action(3.9). First, it is non-local. And the physical origin is still mysterious to us. At first glance one may think winding modes can cause such non-locality. But from the above calculation we see that the effect of these twisted sectors is to induce the infinite summation in eq. (3.7) and thus only leaving trace in the necessity to use an integral in eq. (3.9). In the null brane case [20], where there are similarly twisted sector contributions, D-instanton action is calculated in [18], which is also an integral but with the integrand local. And we see that the non-locality is very peculiar to Misner space whose singularity is spacelike.

It was shown in [21] by Hashimoto and Sethi that the gauge theory on the D3-branes in the null brane [20] background is noncommutative, thus also non-local. What is interesting in their model is that they observe that upon taking some decoupling limit, the noncommutative field theory provides a holographic description of the corresponding time-dependent closed string background (see also [22]). Whether some decoupling limit [23] exists in our case is worth exploring.

Second, notice that the argument in the action (3.9) is complexified, which is a peculiar property of some time-dependent backgrounds. And it is also a crucial ingredient in our following treatment of instability of Misner space and of the branes therein. Complexified arguments also make appearance in the study of other singularities (see for example [24, 25]).

4. D-instantons probing tachyon deformed Misner universe

We go on to deal with the case with winding string tachyon condensates turned on [13]. Take anti-periodic boundary conditions around the θ circle in (2.4). In the regime

$$\gamma^2 T^2 \leq l_s^2, \tag{4.1}$$

some winding closed string modes become tachyonic which signals the instability of the spacetime itself. These modes grow and deform the spacetime. It was speculated in [13] that the regime (4.1) will be replaced by a new phase with all closed string excitations lifted.

D-instantons feel the change in the geometry through its coupling to the metric. It was shown by Douglas and Moore in [26] that the leading effect of tachyons on the euclidean orbifolds is to induce a FI-type term in the D-brane potential. This effect comes from the disk amplitude with one insertion of the twisted sector tachyon field at the center and two open string vertex operators at the boundary. With a detailed analysis of the full quiver gauge theory, which provides a description for D-branes on the orbifolds, they combine the FI term with the Born-Infeld action and the kinetic energies of the hypermultiplets, and then integrate out the auxiliary D-fields in the vectormultiplet, to find that the effect of the twisted sector fields is to add a term in the complete square. In our case, we are dealing with a lorentzian orbifold which is more subtle than its euclidean cousin. But to study the D-instanton theory, we can perform a wick rotation to go to the euclidean case, where the result of [26] will be consulted, and finally we get schematically

$$S = -\frac{1}{g^2}\int_0^{2\pi}\frac{d\sigma}{2\pi}\left[\, X^+(\sigma + i2\pi\gamma)X^-(\sigma) - X^-(\sigma - i2\pi\gamma)X^+(\sigma) - U(\sigma)\,\right]^2 . \tag{4.2}$$

The detailed form of $U(\sigma)$ is not important in the following treatment where we require only the existence of such a non-zero term. There may be some subtlty in the above wick rotation which deserves further clarification. And the above D-instanton action can also be thought of as coming from a time-like T-dual [27] of a more controllable system with D-particles on an euclidean orbifold.

The vacuum condition becomes now

$$X^+(\sigma + i2\pi\gamma)X^-(\sigma) - X^-(\sigma - i2\pi\gamma)X^+(\sigma) = U(\sigma). \tag{4.3}$$

And the leading effect of the tachyons is to make the geometry noncommutative

$$[X^{+}(\sigma), X^{-}(\sigma)] = U(\sigma). \tag{4.4}$$

The above approximation essentially sets $\gamma = 0$. And we know that the geometry corresponding to $\gamma = 0$ is just the flat space without any boost identification, so one may think this case can not teach us much about Misner space. But we note that the term $U(\sigma)$ encodes information peculiar to Misner space. From our experiences for other better understood tachyons [28, 10, 11], we can think this way: the nontrivial boost identification, plus the anti-periodic boundary condition for fermions, first cooks some closed string tachyons. Then these tachyons condense. For these stages we cannot say anything new in the above formalism. We intend only to explore how subsequently spacetime geometry is modified by these tachyon condensates, taking into account the fact that the tachyons couple to the metric. At this stage, the process is driven by the tachyon source while the nontrivial boost identification is not essential, thus we can use the above approximation $\gamma = 0$.

To see the picture more clearly, let's go to the (T, θ) frame. We can model the geometry by choosing

$$e^{\gamma\theta} T e^{-\gamma\theta} = aT, \tag{4.5}$$

with a some constant. This makes a fuzzy cone, where the deviation of a from unity measures the fuzziness of the geometry. And the noncommutative relation (4.4) reads now

$$T^2 \left(a - \frac{1}{a}\right) = 2U. \tag{4.6}$$

We see from the above equation that in the two asymptotic regions $T \to \pm\infty$, a goes to unity, thus T commutes with θ and conventional geometric notion works well. But as T goes to zero, a deviates more and more from unity. Thus spacetime becomes more and more fuzzy. At the origin $T = 0$, a diverges, and the conventional notion of geometry breaks down totally. Thus the original spacelike singularity is removed.

4.1 Comparison with McGreevy and Silverstein' s nothing phase

It is also interesting to compare our result with that of [13] (see also [29]) which actually motivated our study. They employ perturbative string methods, working on the world sheet using techniques from Liouville theory. Here we will intend to propose a non-perturbative formulation of the theory, and the emerging picture is in fact consistent with their work[1] They read from their 1-loop partition function that the volume of the time direction is truncated to the region without closed string tachyon condensates, providing evidence for previously expected picture that closed string tachyon condensation lifts all closed string degrees of freedom, leaving behind a phase of "Nothing". In our formalism, we can say

[1] We give literally different answer to the question: can time start or end by turning on such closed string tachyons, where we employ different interpretation of the question. They say yes [13] where they mean conventional aspect of time breaks down in some region. And we say no having in mind that information can still be transferred from infinite past to infinite future.

more about this "Nothing Phase". Although ordinary concepts of spacetime break down, we can still model such region by some non-commutative geometry. Although closed string degrees of freedom cease to exist in such region, it is nevertheless possible to formulate the theory with open string degrees of freedom. And we expect the D-instanton matrix action (4.2) can serve this role. It seems that matrix models have the potential to say more about closed string tachyons, who are known as killers of closed string degrees of freedom, as open string tachyons did for open string degress of freedom.

Recently it was also found [39] that near some null singularities, the usual supergravity and even the perturbative string theory break down. Matrix degrees of freedom become essential and the theory is more suitably described by a Matrix string theory. Such non-abelian behavior seems intrinsic for singularities.

4.2 A model for big crunch/big bang transition

The whole picture of the resulted spacetime after tachyon condensation is that of two asymptotically flat region, the infinite past and infinite future, connected by some fuzzy cone. And although conventional concept of time breaks down, there is still causal connection between the infinite past and infinite future. This fact is cosmologically attractive.

An alternative to inflation is proposed in [30], where they considered the possibility that the big bang singularity is not the termination of time, but a transition from the contracting big crunch phase to the expanding big bang phase. The horizon problem is nullified in this scenario, and other cosmological puzzles may also be solved in this new framework. Unfortunately it is generally difficult to get a controllable model for such a scenario. From the above discussion, we see that the tachyon deformed Misner universe serves as a concrete model for such big crunch/big bang transition [30].

4.3 Remarks on space-time noncommutativity

Note that the fuzzy cone condition (4.5) is just a statement that space and time do not commute near the spacelike singularity. This leads to the stringy spacetime uncertainty relation, which was suggested by Yoneya and Li to be a universal characterization of short distance structure for string and D-brane physics [31]. Here their idea is realized in a time-dependent background and we can compare with Hashimoto and Sethi's realization [21] of time-dependent space-space noncommutativity. In their case, the noncommutativity can be traced back to the presence of background B field, which is well understood [32]. For Misner space the noncommutativity all comes from the violent fluctuations of the geometry near the singularity, and this needs further study.

Space-time noncommutativity is also interesting for cosmology, since it leads to coupling between inflation induced fluctuations and the background cosmology thus may produce transplackian effects. This subject is much explored in the literature [33], where it was pointed out that short distance dispersion relations may be modified, and non-gaussianity, anisotropism and the running of spectual index can be explained. But usually for lack of concrete models, discussions are made generally. In the model of tachyon deformed Misner universe, more detailed questions can be asked.

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5. A possible holographic description of Misner universe

Now we ask the question: how much information about the Misner space is encoded in the action (3.9) ? In fact, the original action (3.6) plus the fermionic part and a chemical potential term proportional to $\mathrm{Tr}\,1$ is proposed in [17] to provide a constructive definition of definition of type-IIB string theory. This so called IKKT matrix theory, is interpreted in [34] as the D-instanton counterpart of the D0-brane matrix theory of BFSS [16]. Fundamental strings and Dp-branes [17, 34, 35] can be constructed from such matrices, and long-distance interaction potentials of BPS configurations computed from such matrices match the supergravity results [36]. We can extend this matrix/string correspondence to the Misner case by making orbifold projections (2.2) on both sides, where obviously the projection commutes with the matrix/string mapping. So we propose that the action (3.9) (plus its fermionic counterpart and possible chemical potential term) provides a holographic description of the Misner universe.

For these 0+0 (or here under orbifold projection, 1+0) dimensional model, time is not put in a priori, but generated dynamically. This may be the underlying reason why our description of change of the structure of time is possible. And it also indicates that these matrix models may have privilege in the description of spacelike singularities and other time-dependent systems.

We go on to construct branes in Misner universe. The equation of motion of action (3.6) is

$$g_{\mu\rho}g_{\nu\sigma}[X^\nu,[X^\rho,X^\sigma]]=0, \tag{5.1}$$

which in components are

$$\begin{aligned}
\big[X^-,\big[X^-,X^+\big]\big]-\big[Y^a,\big[X^-,Y^a\big]\big]&=0,\\
\big[X^+,\big[X^+,X^-\big]\big]-\big[Y^a,\big[X^+,Y^a\big]\big]&=0,\\
\big[X^+,\big[Y^a,X^-\big]\big]+\big[X^-,\big[Y^a,X^+\big]\big]&=0.
\end{aligned} \tag{5.2}$$

As above we expand the matrices in the β basis (3.3), and make the transformation (3.8), and choose $Y(\sigma)$ to constant. Thus we get the classical solutions

$$\begin{aligned}
&\int\frac{d\sigma}{2\pi}\,\mathrm{Tr}\,X^-(\sigma)\Bigg(X^+(\sigma+i4\pi\gamma)X^-(\sigma+i2\pi\gamma)-X^-(\sigma)X^+(\sigma+i2\pi\gamma)-\\
&\qquad\qquad -X^+(\sigma+i2\pi\gamma)X^-(\sigma)+X^-(\sigma-i2\pi\gamma)X^+(\sigma)\Bigg)=0,\\
&\int\frac{d\sigma}{2\pi}\,\mathrm{Tr}\,X^+(\sigma)\Bigg(X^-(\sigma-i4\pi\gamma)X^+(\sigma-i2\pi\gamma)-X^+(\sigma)X^-(\sigma-i2\pi\gamma)-\\
&\qquad\qquad -X^-(\sigma-i2\pi\gamma)X^+(\sigma)+X^+(\sigma+i2\pi\gamma)X^-(\sigma)\Bigg)=0.
\end{aligned} \tag{5.3}$$

which are unusually integral equations.

We can define

$$L(\sigma)\equiv X^+(\sigma+i2\pi\gamma)X^-(\sigma)-X^-(\sigma-i2\pi\gamma)X^+(\sigma), \tag{5.4}$$

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which appears repeatedly in this note. Note that $L(\sigma) = 0$ is just the vacuum (3.11). Consider in the moduli space (3.22) a special configuration

$$X^+(\sigma) = X^-(\sigma) = \begin{pmatrix} t_1 & & & \\ & t_2 & & \\ & & \ddots & \\ & & & t_N \end{pmatrix},$$
$$Y^a = 0, \tag{5.5}$$

where $t^{(i)}$'s are constants. In the large-N limit, we see from the corresponding classical trajectory

$$T = t, \qquad \theta = 0, \qquad Y^a = 0 \tag{5.6}$$

with parameter t, that it is just a D0-brane [17]. And the D-instanton action at this point of the moduli space reproduces the D0-brane action of the BFSS matrix theory [17].

It seems strange that D0-branes emerge this way from a D-instanton matrix model which is directly related to the type-IIB theory. In flat space, these D0-branes are supersymmetric and stable.[2] And in Misner space they will also not decay for their seemingly wrong dimension. The IKKT proposal [17] is that since type-IIA string theory is related to type-IIB theory by T-duality, in some regions of the type-IIB moduli space, the type-IIA theory can emerge as a more suitable description. And the existence of these D0-branes will be considered as manifestation of duality.

It is pointed out in [15] that D0-branes in Misner universe are actually unstable, they are subject to open string pair creation. Since we regard our instanton action to be a second quantized description of Misner universe, such phenomena should be reproduced.

Let's make some small perturbation around the D0-brane (5.5)

$$X^+(\sigma) = \begin{pmatrix} t_1+\delta t_1 & & & \\ & t_2 & & \\ & & \ddots & \\ & & & t_N \end{pmatrix},$$
$$X^-(\sigma) = \begin{pmatrix} t_1 & & & \\ & t_2 & & \\ & & \ddots & \\ & & & t_N \end{pmatrix},$$
$$Y^a = 0, \tag{5.7}$$

with $\delta t^{(1)} = \epsilon\sigma$. Now the action becomes

$$\begin{aligned} S &= -\frac{1}{g^2}\int \frac{d\sigma}{2\pi} t_1^2[\delta t_1(\sigma + i2\pi\gamma) - \delta t_1(\sigma)]^2 \\ &= \left(\frac{2\pi\gamma t_1}{g}\right)^2 \epsilon^2. \end{aligned} \tag{5.8}$$

[2]Brane charges are more subtle in the IKKT matrix theory than that of BFSS [38]. Some proposals were made in [35] for Dp-branes with p odd. But for p even, (linear combinations of) the matrices commute with each other, leaving no room for constructing central charges along the lines of [38].

Note the sign change above, which originates from the complexified arguments in the integrand. And perturbations of other eigenvalues give similar results.

To understand the above action, consider a quantum mechanical system

$$S = \int dt \left[\frac{1}{2}\dot{X}^2 - U(X)\right]. \tag{5.9}$$

With $U(X) = -\frac{1}{2}kX^2$ and $k > 0$, it is just a particle moving in an inverted harmonic potential. Seemingly the particle can not stay static. It will roll down the potential. When the potential term dominates the whole action, we go over to the action (5.8). And accordingly the D0-branes are unstable. Worse still, the action is even not bounded from below, which makes it impossible to define a first quantized vacuum. This fact has already been noticed in [6] in their study of perturbative string theory of Misner universe.

Such kind of inverted harmonic potential also appears in $c = 1$ matrix model, and there closed string emission from unstable D0-branes is described by a matrix eigenvalue rolling down such a potential [37]. In our formalism, description of such dynamical processes is intrinsically subtle, where technically the difficulty stems from the fact that we do not have kinetic terms for the matrix eigenvalues. But we can understand from the path integral point of view that, the smaller the euclidean action $S_E = -S$, the more the configuration contributes to the whole amplitude. And if we start with the D0-brane (5.5), quantum fluctuations will generally destroy this configuration, driving the system to more probable configurations with larger t_i, making the brane effectively "roll down" the potential. In the large-N limit, this corresponds to the phenomena that the unstable D0-branes emit closed strings and/or open string pairs [37].

For the background geometry

$$\begin{aligned} X^+(\sigma) &= x^+ I_{N\times N}, \\ X^-(\sigma) &= x^- I_{N\times N}, \\ Y^a(\sigma) &= y^a I_{N\times N}, \end{aligned} \tag{5.10}$$

we can likely make a perturbation

$$\begin{aligned} X^+(\sigma) &= \begin{pmatrix} x^+ + \delta x^+ & & \\ & x^+ & \\ & \ddots & \\ & & X^+ \end{pmatrix}, \\ X^-(\sigma) &= x^- I_{N\times N}, \\ Y^a(\sigma) &= y^a I_{N\times N}. \end{aligned} \tag{5.11}$$

And similarly we get an inverted harmonic potential with δx^+ a linear perturbation, thus the same instability arises, which is consistent with what is found in perturbative string theory in Misner space [6, 7] which states that the vacua is unstable while winding strings are pair produced as a consequence of the singular geometry in analogy with the Schwinger effect in an external electric field. This is a tunnelling process, matching precisely our description via D-instantons. And in this matrix framework, we can see that the instabilities of the geometry and the branes have essentially the same origin. Both can be interpreted as matrix eigenvalues "rolling down" a unbounded-from-below potential.

Next let's discuss the D-strings. It is easy to see from (5.3) that $L(\sigma) = constant$ is a classical solution. The Minkowski limit $\gamma \to 0$ of $L(\sigma)$ is just the commutator $[X^+(\sigma), X^-(\sigma)]$, and in this limit $L(\sigma) = constant$ becomes the familiar result in matrix theory

$$[X^+(\sigma), X^-(\sigma)] = i\mathcal{F}^{+-} I_{N\times N}, \tag{5.12}$$

with $\mathcal{F}^{+-}$ some non-zero constant. And there in the large-N limit, it represents D-strings [17] or some non-marginal bound states of D-strings with D-instantons [34].

Here the solution corresponding to a D-string is

$$\begin{aligned} X^+(\sigma) &= \frac{L^+}{\sqrt{2\pi N}} q, \\ X^-(\sigma) &= \frac{L^-}{\sqrt{2\pi N}} p, \\ Y^a &= 0, \end{aligned} \tag{5.13}$$

with L^+, L^- some large enough compactification radius, and the $N \times N$ hermitian matrices $0 \le q, p \le \sqrt{2\pi N}$ satisfying

$$[q, p] = I_{N\times N}, \tag{5.14}$$

which is obviously only valid at large-N. Note also the omitted i in our convention in contrast to usual notion.

These D-strings are also unstable [15], and the interpretation in matrix theory is essentially the same as for D0-branes and the geometry. We add some small perturbations, say change q_{11} to $q_{11} + \epsilon\sigma$, and the real part of the action becomes now

$$S_{pert} = S_{D1} + \left(\frac{2\pi\gamma p_{11}}{g}\right)^2 \epsilon^2, \tag{5.15}$$

leading to the "rolling" behavior of the matrix elements and thus D-string's emitting open or closed strings.

The universality of the interpretation of instabilites of D0- and D1-branes provides further evidence that D0-branes do not decay for their "wrong dimensionality" and the region around (5.5) has a more suitable description as type-IIA string theory.

Obviously more efforts are needed to figure out the details of the string emission, such as the spectrum and emission rate which have already been calculated in perturbative string theory [7, 15]. The matrix formalism has the potential advantage to treat more precisely the backreaction of the emitted strings as we have exampled in section 4.

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